2. Set $\mathbf{v}_1 = \mathbf{x}_1$ and compute that $\mathbf{v}_2 = \mathbf{x}_2 - \frac{\mathbf{x}_2 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\mathbf{v}_1 = \mathbf{x}_2 - \frac{1}{2}\mathbf{v}_1 = \begin{bmatrix} 5 \\ 4 \\ -8 \end{bmatrix}$. Thus an orthogonal basis for W is $\left\{ \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \\ -8 \end{bmatrix} \right\}$.

4. Set $\mathbf{v}_1 = \mathbf{x}_1$ and compute that $\mathbf{v}_2 = \mathbf{x}_2 - \frac{\mathbf{x}_2 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\mathbf{v}_1 = \mathbf{x}_2 - (-2)\mathbf{v}_1 = \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}$. Thus an orthogonal basis for W is $\left\{ \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix} \right\}$.

8. Since $\| \mathbf{v}_1 \| = \sqrt{50}$ and $\| \mathbf{v}_2 \| = \sqrt{54} = 3\sqrt{6}$, an orthonormal basis for W is

$$\left\{ \frac{\mathbf{v}_1}{\| \mathbf{v}_1 \|}, \frac{\mathbf{v}_2}{\| \mathbf{v}_2 \|} \right\} = \left\{ \begin{bmatrix} 3/\sqrt{50} \\ -4/\sqrt{50} \\ 5/\sqrt{50} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix} \right\}.$$

12. Call the columns of the matrix $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ and perform the Gram-Schmidt process on these vectors:

$$\mathbf{v}_1 = \mathbf{x}_1$$

$$\mathbf{v}_2 = \mathbf{x}_2 - \frac{\mathbf{x}_2 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\mathbf{v}_1 = \mathbf{x}_2 - 4\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$\mathbf{v}_3 = \mathbf{x}_3 - \frac{\mathbf{x}_3 \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1}\mathbf{v}_1 - \frac{\mathbf{x}_3 \cdot \mathbf{v}_2}{\mathbf{v}_2 \cdot \mathbf{v}_2}\mathbf{v}_2 = \mathbf{x}_3 - \frac{7}{2}\mathbf{v}_1 - \frac{3}{2}\mathbf{v}_2 = \begin{bmatrix} 3 \\ 3 \\ 0 \\ -3 \\ -3 \end{bmatrix}$$

Thus an orthogonal basis for W is $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 0 \\ -3 \\ -3 \end{bmatrix} \right\}$.

16. The columns of Q will be normalized versions of the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ found in Exercise 12. Thus

$$Q=\begin{bmatrix} 1/2 & -1/2\sqrt{2} & 1/2 \\ -1/2 & 1/2\sqrt{2} & 1/2 \\ 0 & 1/\sqrt{2} & 0 \\ 1/2 & 1/2\sqrt{2} & -1/2 \\ 1/2 & 1/2\sqrt{2} & 1/2 \end{bmatrix}, R=Q^T A=\begin{bmatrix} 2 & 8 & 7 \\ 0 & 2\sqrt{2} & 3\sqrt{2} \\ 0 & 0 & 6 \end{bmatrix}$$

19. Suppose that $\mathbf{x}$ satisfies $R\mathbf{x} = \mathbf{0}$; then $Q\,R\mathbf{x} = Q\mathbf{0} = \mathbf{0}$, and $A\mathbf{x} = \mathbf{0}$. Since the columns of A are linearly independent, $\mathbf{x}$ must be $\mathbf{0}$. This fact, in turn, shows that the columns of R are linearly indepedent. Since R is square, it is invertible by the Invertible Matrix Theorem.